Section- A (MCQ's)

Q.1: Choose the correct answer for each formthe given options:

(i) If $f(x)-2x$, $8(x)=1/2x$ and $fog(x)=6$ then x is:

(a) 4 (b) 12 (c) 6 (d) 2

(ii) $\lim_{x\to 0}\frac{\cos x-1}{x}=$ _____

(a) 1 (b) 0 (c) -1 (d) 2

(iii) If $f(x)=\frac{3x-4}{2}$, then the valoe of $f'(x)$ is:

(a) $\frac{3x+2}{4}$ (b) $\frac{2y+3}{4}$ (c) $\frac{2x+4}{3}$ (d) None of these

(iv) If $y=ax$ (where $a\in\Re^+$ and if $0<a<1$ then a is _____

(a) Decreasing function
(b) Increasing function
(c) Exponenetial function
(d) Constant function

(v) The centraid G of a ftrinagle divides each median in the ratio _____

(a) 1:2 (b) 2:1 (c) -2:1 (d) 1:1

(vi) If $m_1=\frac{1}{2}$ and $m_2=-2$ then $\tan\theta$ will be:

(a) ∞ (b) 10 (c) 45 (d) 90

(vii) The distance of the point (6, -2) from the line $3x-4y+4=0$ is _____

(a) -5 units (b) 5 units (c) -6 units (d) -6 units

(viii) $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=$ _____

(a) f(x') (b) f(h') (c) f'(x) (d) f'(x+h)

(ix) If the slope of the line joining the points (6, 3) and (4, k) is -3, then k is ____

(a) 0 (b) 9 (c) -9 (d) 3

(x) If measure of the angle between the two lines $ax^2 - 2hxy + hy^2 = 0$ is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ then the two line are _____

(a) Parallel (b) Coineident (c) Perpendicualr (d) Neither

(Xi) $\frac{d}{dx}\sin^{-1}u =$ _____

(a) $\frac{1}{\sqrt{1-u^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1-n^2}}$ (d) $\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

(xii) If $y=\sqrt{\sin 5x}$, then $\frac{dy}{dx}=$ _____

(a) $\frac{5\cos 5x}{5}$ (b) $\frac{5\cos 5x}{2\sqrt{\sin 5c}}$ (c) $\frac{1}{2\sqrt{\sin 5x}}$ (d) $\frac{-\sin 5x}{2\sqrt{\cos 5x}}$

(xiii) y = f(x) is increasing if:

(a) $\frac{dy}{dx}<0$ (b) $\frac{dy}{dx}=0$ (c) $\frac{dy}{dx}>0$ (d) $\frac{dy}{dx}=\infty$

(xiv) If a = 3i _ xj + 2k and b = 4i + 3K are perpendicular then x = _____

(a) 18/7 (b) 2/3 (c) 3 (d) 6

(xv) $x^2 + y^2 - 2gx - 2fy + f^2 = 0$ rerp[esent a circle towching the _____

(a) x-axis (b) y-axis (c) origin (d) boht he axis

(xvi) $y^2 = 4ax$ represents a curve sysmeetrical about the _____

(a) x-axis (b) origin (c) y-axis (d) both the axis

Note: Solve any TEN of the following questions. Each question carries 05 marks.

Q.2: Show that the sequence $1-\frac{3}{4}+\frac{9}{16}-\frac{27}{64}+$ ..... is divergent.

Q.3: Show that $\lim_{x\to\infty}\frac{\ln(1+a^x)}{x}=1$

Q.4: Show that the point $(\frac{13}{4},\frac{9}{4})$ is the centre of the circle circumscruibing the triangle whose vertices are (2,1), (5,2) and (3, 4) respectively.

Q.5: Find the combined equation of the pair of lines through the origin which are perpendicular to the lines $2x^2 - 5xy + y^2=0$.

Q.6: If $x^2=y^2$ find $\frac{dy}{dx}$

Q.7: Calculate log10(10.10) given that $\log_e e = 0.4343$.

Q.8: The curve $y=x^3 - 3x^2 - 9x + k$, where k is constant. The curve has a minimum point of the x-axis . Find the value of k, Also find the coordinates of the maximum point of the curve.

Q.9: Use the substitution $x=\sin^2\theta$. Find the exact value of $\int_6^{1/4}\sqrt{\frac{x}{1-x}}\,dx$

Q.10: Prove analytically a normal to a circle passes through the centre of the circle.

Q.11: Find the equation of the ellipese having the origin as centre, are focus at the point (3,0) and the corresponding directrix x = 8.

Q.12: Show that hte eccentricities e_1 and e_2 of the conjugate hyperblas satisfy the relation . $e_1^2 + e_2^2 = e_1^2e_2^2$

Q.13: Let $f(x)=\frac{7x+4}{(2x+1)(x+1)^2}$ express f(x) in partial fractions.

Q.14: The vertices of a triangle are the points A:(1,2 1) (-3, 1,1) , C:(1,-1,2). Forces of magnitude 3 and 5 act at a along $\vec{AB}$ and $\vec{AC}$ Find their resultant.

Section-C

Note: Attempt any THREE of the following questions.

Q.15(a): Find the derivative of f(x) = sec2x by first principle.

(b) Use integration by parts to show that $\int_2^4 \ln x\,dx = 6\ln 2 - 2$.

Q.16(a): A is the mide piont of the segment bounded by (-4,4) and (2, 2) B is a point at $\frac{3}{5}$ of the distance from (5,3) to (-3, 2). Find the equation of $\overline{AB}$.

Q.17(a) Derive the eqaution fo striaght line in the normal form x .

(b): Evaluate f cotx, cosec x dx.

Q.18(a): If $y = f(x) = ae^x + be^{-x} + ce^{-}$ for all x show that $\frac{d^2y}{dx^2} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

(b) A cruve is such that $\frac{dy}{dx} = 2x^2 - 5$. Given that point (3, 8) lies on the curve, Find the equatino of the curve.

Section-A

Mulitple Choice Questions (MCQ's)

Q.1: Choose the correct answer for each form th

(1) A point where a function 'f' changes form an increasing function to decreasing function is called:

(a) A point of maximum value (b) A point of minimum value
(c) point of zero value (d) None of these

(2) $\int x \cos x \, dx$

(a) $x \sin x + \cos x + c$ (b) $x \sin x + \cos x + c$
(c) x sion x – cos x + c (d) x cos x – sin x + c

(3) If (x_1, y_1) and $(x_2 - y_2)$ are end point of the diameter then centre of circle is:

(a) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ (b) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(c) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ (d) $(x_1 + x_2, y_1 + y_2)$

(4) If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are any two points on parabola $y^2 = 4ax$ then eq : of the chord PQ will be:

(a) $(y + y_1)(y_1 - y_2) = 4a \cdot x - x1$ (b) $(y - y_1)(y_1 + y_2) = 4a(x - x_1)$
(b) $(y - y_1)(y_1 + y_2) = 4a(x - x1)$ (d) $(y + y_1)(y_1 + y_2) = 4a(x + x_1)$

(5) The vector product of a vector by itself is always:

(a) 1 (b) -1 (c) greater than 1 (d) 0

(6) $\lim_{x \to 0} (1 + 3x)^{2/3}$

(a) e^3 (b) e^{-6} (c) e^6 (d) e

(7) The point (-3,0) (0,0) and (5,0) are:

(a) vertices of triangle (b) collinear point (c) Non-collinear (d) None

(8) The x- intercept of the line $3x + 4y - 7 = 0$

(a) $-\frac{7}{3}$ (b) $\frac{7}{3}$ (c) $\frac{7}{4}$ (d) $-\frac{7}{4}$

(9) The slope of the curve $y = 3e^{2x}$ at $x = 0$ is equal to :

(a) 3 (b) 6 (c) 5 (d) 2

(10) Maximum values of f(x) are minimum value of :

(a) f (x) (b) –f (c) f'(x) (d) –f' (x)

(11) $\int (ax + b)^n dx = ?$

(a) $\frac{(ax + b)^{n+1}}{an + 1}$ (b) $\frac{(ax + b)^{n-1}}{an + 1}$ (c) $\frac{(ax + b)^{n+1}}{an + 1}$ (d) $\frac{(ax + b)^{n+1}}{a(n + 1)}$

(12) Two circles are said to be concentric if they have:

(a) same radii (b) same centre

(c) same diameter (d) different centre

(13) The line $y = mx + c$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if

(a) $c = a/m$ (b) $c^2 = a^2 m^2 + b^2$ (c) $c^2 = a^2m^2 - b^2$ (d) $c^2 = a^2 (1+m^2)$

(14) What does scalar triple product gives:

(a) Area of square (b) Volume of cylinder (c) Volume of square

(d) Volume of parallelepiped

(15) If $\lim \frac{a^x - 1}{x} = ?$

(a) $\ln a$ (b) $\ln x$ (b) $a \ln x$ (d) $1/a \ln a$

(16) If two non vertical lines are parallel their inclinations and their slopes are:

(a) Perpendicular (b) Zero (c) Not equal (d) Equal

(17) Which one is two intercept form:

(a) $\frac{x}{a} + \frac{y}{b}$ (b) $\frac{x}{a} \cdot \frac{y}{b}$ (c) $\frac{x}{a} + \frac{y}{b} = 0$ (d) $\frac{x}{a} + \frac{y}{b} = -1$

(18) (-2, 5), (4, 1) and (5, 4) are the vertices f triangle its centroid is :

(a) $\left(\frac{7}{3}, \frac{8}{3}\right)$ (b) $\left(\frac{11}{3}, \frac{8}{3}\right)$ (c) $\left(\frac{7}{3}, \frac{10}{3}\right)$ (d) (0,1)

(19) If $\int \pi d\theta = ?$ (a) $\pi + c$ (b) $\pi\theta + c$ (c) $\theta + c$ (d) $\pi/3 + c$

(20) $\int [f(x)]^n f(x)\, dx = ?$

(a) $\frac{[f(x)]n+1}{n+1}$ (b) $\ln [f(x)]^{n+1}$ (c) $\ln [f(x)]^n$ (d) All of these

(21) The general equation of he circle whose centre lies on x -axis:

(a) $x^2 + y^2 + 2fy + c = 0$ (b) $x^2 + y^2 - 2gx + c = 0$

(c) $x^2 + y^2 + 2gx + c = 0$ (d) $x^2 + y^2 + 2gx - c = 0$

(22) The point P(x1, y1) lies outsides the circle x2 + y2 + 2g x + fy + c = 0 if

(a) $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$ (b) $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

(c) $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$ (d) $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

(23) If $a = (2, -1, 2)$ $B = (-4, 1, -1)$ then $\overline{AB} = ?$

(a) $-6i - 2j + 3k$ (b) $-6i + 2j + 3k$ (c) $6i + 2j - 3k$

(d) $6i - 2j - 3k$

(24) $\lim_{x \to \infty} ex = ?$ (a) 1 (b) 0 (c) $\ln x$ (d) ∞

(25) Two lines $5x - 3y + 8 = 0$ and $10x - 6y - 7 = 0$ are

(a) Parallel (b) Perpendicular (c) Coincident (d) None of these

(26) $\lim_{x \to 0} \frac{x^2 - 1}{x^2 - x}$ (a) 1 (b) 2 (c) x (d) ∞

(27) The equation of the x –axis itself as

(a) $x = 0$ (b) $y = 0$ (c) $y = x$ (d) None of these

(28) Three lines are concurrent of $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = ?$

(a) 1 (b) -1 (c) $\pm$ (d) 0

(29) If the point on the graph $y = f(x)$ where $f(x)$ is neither increasing nor decreasing and $f(x) = 0$ then that point is called:

(a) Maximum point b) Minimum point

(c) Stationary point d) Turning point

(30) I $d/dx [\phi(x)] = f(x)$ then function $\Theta(x)$ is called:

(a) Derivativeof f(x) (b) Antiderivatve of f(x) (c) inverse of f(x)

(d) Definite integral

(31) The centre of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$,

(a) (g, -f) (b) g, f) (c) (g , f) (d) (g, -f)

(32) $\int f(x)\,dx \; \forall \; a,b \in R, a \leq b$ is called:

(a) Definite integral (b) Indefinite integral (c) Simple integral (d) None

(33) If a = [P, 1, 0], b = [1, 1,3] and c=[2, 1,-2] are coplanar then P = ?

(a)3/4 (b) 4/5 (c) 6/5 (d) 8/5

(34) If the point p (x, y), x is called.

(a0 ordinate (b) abscissa (c) pole (d) None of these

(35) If $a \neq 0, c \neq 0$ and b = 0 then ax + by + c = 0 represents a line:

(a) Parallel to x –axis (b) Parallel to y –axis (c) Passing through the origin

(d) bisecting 1st and 3rd quad

(36) $f(x) = x^3 + 4x^2 + 3x + 9$ then $f'(x) = ?$

(a) $6x + 8$ (b) $6x^2 + 8x$ (c) $6x - 4$ (d) $3x + 9$

(37) Two non-zero vectors $\underline{U}$ and $\underline{V}$ are parallel if:

(a) u : v = 0 (b) u x v = 0 (c) both (a) and (b) (d) None of these

(38) If we put x = a + h in function f(x) then the limit of f(x) as $x \to a$ is:

(a) the limit of f(x) as $h \to 0$ (b) limit of f(a + h) as $x \to a$ is

(c)the limit of f(a + h) as $h \to a$ (d) limit of f(a + h) as $\to 0$

(39) y – co- ordinate of any point on the x-axis

(a) zero (b) one (c) y (d) x

(40) The point of concurrency of there lines 2x – 5y = 0, 3x – 17y = 0 and x + 6x = 0 is

(a) (-1, -1) (b) (0, 0) (c) (1, 1) (1/2, ½)

(41) The lines lying on the same plane are called:
(a) Collinear lines (b) coplanar lines (c) Concurrent lines
(d) Coincident lines

(42) If $y = e^{2x} - 1$ then dy/dx is euqla to :
(a) $2e^{2x} - 1$ (b) $2e^{2x}$ (c) 2ex (d) None of these

(43) The function f(x) = sin x as maximum value :
(a) 0 (b) 1 (c) -1 (d) $-\infty$

(44) The inverse operator of d/dx is
(a) $\int dx$ (b) dx (c) Δ^2 (d)none of these

(45) If the centre of circle $x^2 + y^2 + ax + by + 2 = 0$ is at (4, -8) the a + b = ?
(a) -4 (b)4 (c) 8 (d) -8

(46) The centre of circle at origin and radius r, h = 0 , k = 0 then
(a) $x^2 - y^2 - r^2 = 0$ (b) $x^2 + y^2 = r^2$ (c) $y^2 - x^2 = r^2$ (d) $x^2 - y^2 = -r^2$

(47) The unit vector I is :
(a) [0, 0, 1] (b) 0, 1, 0] (c) 1, 0, 0] (d) None of these

(48) If $\rho(\theta) = (x, y)$ when $= 180°$, then (x, y) = _____
(a) (0, 1) (b) (0, -1) (c) (-1, 0) (d) (1, 0)

(49) Find θ, if s = 20 cm and r = 2m
(a) 0.1 radian (b) 10 radian (c) 0.1 degree (d) 10 degree

(50) cos 2θ = _____
(a) $2\cos^2\theta - 1$ (b) $\cos^2\theta - \sin^2\theta$ (c) $1 - 2\sin^2\theta$ (d) All of these

Section-B

Note: Solve any SIX of the following questions . Each question carries 05 marks:

Q.2 Find the limit of $\lim_{x \to 0} \frac{1 - \cos 2x}{1 - \cos 3x}$

Q.3 If $f(x) = (\tan x)^{\cos x + \sin x}$, find f(x).

Q.4 Find the equation of a line through the intersection of the lines $5x - 6y = 1$, $3x + 2y + 5 = 0$ and perpendicular to $5y - 3x - 11 = 0$

Q.5 Fine the extreme value of the function f given by $f(x) = x(x-1)(x-2)$ for all $x \in R$.

Q.6 Evaluate $\int \frac{3x + 2}{x - 1} dx$

Q.7 Find the equation of the circle having (-5, 6) and (3, -4) the end points of a diameter.

Q.8 Find the equation of the tangent and normal to the hyperbold $2x^2 - 3y^2 = 1$ and at the point $(\sqrt{2}, 1)$

Q.9 The x – intercept of a line is the reciprocal of its y – intercept, The line passes through (2, -1). Find its equation.

Q.10 If a function, given by $y = f(x)$, $\sqrt{x^2 + y^2} = \ln(x^2 - y^2)$ find dy/dx , where if exist.

Section-C

Note: Solve any TWO of the following question.

Q.11 (a) If $y - \tan 2x$, find dy/dx from definition.

(b) Evaluate $[2i + 3j - l, 3k, j]$

Q.12 (a) The measure of the angle form a line l_1 with slope -1/2 to a line l2 is $3\pi/4$. Find the slope of l_2.

(b) Find $\int x^2 e^x dx$,

Q.13 (a) If $y = f(x) = ae^x - be^{2x} + ce^{3x}$ for all $x \in R$ wher a, b, $c \in R$, show that

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{d^2y}{dx} - 6y = 0$$

(b) Find $\lim_{x \to \infty} \frac{\sqrt{x^4 - 4}}{(x^2+1)^{1/3}}$

Section-A (MCQ's)

Q.1 Choose the correct answer for each from the given options:

(I) $\int \frac{ax+b}{ax^2+2bx+c}dx =$ _____

(a) $2\ln|ax^2+2bx+c|+c$ (b) $\ln|ax+b|c$ (c) $\ln|ax^2+2bx+c|+c$

(D) $\frac{1}{2}\ln|ax^2+2bx+c|+c$

(ii) If $A=\int_0^n \sqrt{1-\cos^2 x}\,dx$, then A = _____.

(a) 0 (b) 2 (c) -2 (d) None of these

(iii) The point of concurrent of altitutdes of a triangle is called _____

(a) Circum-centre (b) In-centre (c) Centroid (d) Orthocentre

(Iv) $ax^2+2hxy+by^2=0$ represent a pair of orthogonal lines If sum of the coefficients of x and y is:

(a) 1 (b) 0 (c) -1 (d) 2

(v) If $90^\circ<\theta<180^\circ$, then slope is:

(a) Undefined (b) Positive (c) Negative (d) Same

(vi) $\lim_{x\to 0}(1-3x)^{\frac{2}{x}} =$ _____

(a) e^3 (b) e^{-3} (c) e^{-6} (d) e^{-3}

(vii) A function $f: A \to B$ is said to be _____, if the range of f is B.

(a) Bijective (b) Surjective (c) Injective (d) None of these

(Viii) $\int \frac{1}{x\ln x}dx =$ _____.

(a) $\ln|x|+c$ (b) $\ln|\ln x|+c$ (c) $\ln x+c$ (d) $\frac{1}{x}\ln x+c$

(ix) The distance between two parallel lines $4x+3y+2=0$ and $4x+3y+8=0$ is:

(a) $\frac{13}{2}$ (b) $\frac{59}{2}$ (c) $\frac{6}{5}$ (d) $\frac{2}{5}$

(x) If the slope of tangent is $\sqrt{3}$, then the angle is:

(a) 30° (b) 60° (c) 45° (d) None of these

(xi) $(1,5) \cap [3,7]$

(a) $(1,7]$ (b) $(3,5)$ (c) $[3,5)$ (d) $(3,5]$

(Xii) If $f(x)=\sqrt{x+2}$ then range of f is:

(a) $(0,\infty)$ (b) $(-\infty,+\infty)$ (c) $(-2,+\infty)$ (d) $[-2,\infty)$

(xiii) If $y = 5e^{3x-4}$, then $\frac{dy}{dx} =$ _____.

(a) $15e^{3x-4}$ (b) $5e^{3x+4}$ (c) $\frac{e^{3x-4}}{3}$ (d) $(3x-4)e^{3x-4}$

(Xiv) The vertex of the parabola $y = 5(x-2)^2 + 7$ is:

(a) (2, 0) (b) (2, -7) (c) (-2, 7) (d) (2, 7)

(xv) The set $S(c, r) = \{px, y): |\ , |cp| = r\}$ contains centre c if:

(a) $r = 0$ (b) $r < 0$ (c) $r > 0$ (d) $r = \infty$

(Xvi) $\int e^{3x}(3\sin x + \cos x)dx =$ _____.

(a) $e^{3x}\sin x + c$ (b) $3e^{3x}\sin x + c$ (c) $e^{3x}\cos x + c$ (d) None fo these

(xvii) If $\vec{u}, \vec{v}, \vec{w}$ are contermious edges of aparallelpiped, then its volume is:

(a) $|\vec{u} \times \vec{v}|$ (b) $[\vec{u}\ \vec{v}\ \vec{w}]$ (c) $\vec{u}, \vec{v}, \vec{w}$ (d) $\vec{u} \times \vec{v} \times \vec{w}$

(xviii) The centre of circle $3x^2 + 3y^2 + 6x + 12y - 7 = 0$ is:

(a) 2, -4) (b) 1, -2) (c) -2, 4) (d) (-1, 2)

Section-B

Note: Solve anyTEN of the following questions. Each question carries 05 marks.

Q.2 Determine the limit of the sequence $\frac{1.2}{3.4}, \frac{3.4}{5.6}, \frac{5.6}{7.8}, \ldots\ldots$

Q.3 Evaluate: $\lim_{x \to c} \frac{x^n - c^n}{x^m - c^m}$

Q.4 Find the point which is equidistant from (2, 1), (5, 2) and (3, 4).

Q.5 Find the equation of Hyperbola with centre at origin, focus at (6, 0) and accentricity is 3.

Q.6 The point (2, – 5) is a vertex of a square one of whose side lies on the line x – 2y – 7 = 0.

Q.7 The gradient of one of the lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. Show that $8h^2 - 9ab = 0$.

Q.8 Find the derivative of $\sqrt{\sin x}$ by first principle.

Q.9 If $y = (\tan t)^{\ln x}$. Find $\frac{dy}{dx}$

Q.10 Find the extreme value of the function f given by $f(x) = (x-1)(x^2-2x)$, $\forall\, x \in R$.

Q.11 Intergrate: $\int 25g \tan^2 5x \, dx$

Q.12 Find the equation of the circle passes through the points (0, 0), (3, 0) and (0, – 4)

Q.13 Find the angle between the positive y–axis and the vector $\vec{a} = \hat{i} + 3\hat{j} + 2\hat{k}$

SECTION – C (DESCRIPTIVE ANSWER)

Q.14 (a) Calculate approximate value of $\cos 61^0$

(b) Intigrate: $\int \frac{2y\,dy}{(y^2+1)(y^2+3)}$

Q.15 (a) Find the equation of the perpendicular bisector of the segment joining (3, 4) and (11, 14).

(b) Find the vertex, focus and equation of of directrix of parabola

$$y^2 - 6y + 8x - 23 = 0$$

Q.16 (a) Find the area of the triangle formed by the lines

$$y - x = 0\,;\, y + x = 0\,;\, x - b = 0$$

(b) Find the equation of the tangents to the circle $x^2 + y^2 = 10$ at the point whose abscissa is 3.